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# Experience from implementation of the new national CRS realisation in Slovakia

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# Agenda

- Legal introduction of the new national CRS realization JTSK03 in Slovakia
- JTSK03
  - History
  - Definition
  - JTSK03 relation to ETRS89
- Problems occurred during JTSK03 implementation
- Conclusions and recommendations



## § Legislation §

- 1<sup>st</sup> April 2011 – **JTSK03** as a new realisation (new reference frame) of the national CRS (S-JTSK) in Slovakia was introduced
- **JTSK03** validation was introduced by acceptance of Amendment 75/2011 Z.z. of UGKK SR Regulation 300/2009 Z.z.
- Amendment 75/2011 Z.z. says:
  - "... actual reference frame of national coordinate reference system S-JTSK is realisation JTSK03."
  - "... Valid JTSKyy realisation has defined relation to national ETRS89 realisation which was computed from and has a homogenous scale with it."





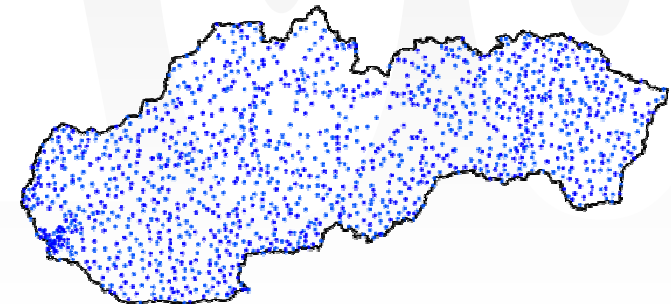
# What is JTSK03?

## JTSK03

### New reference frame of national CRS S-JTSK

- new realisation of old national CRS S-JTSK
- planar 2D system
- JTSC03 is based on relation to ETRS89
- all points which have coordinates determined in ETRS89 have also coordinates in JTSC03 and this is valid vice versa

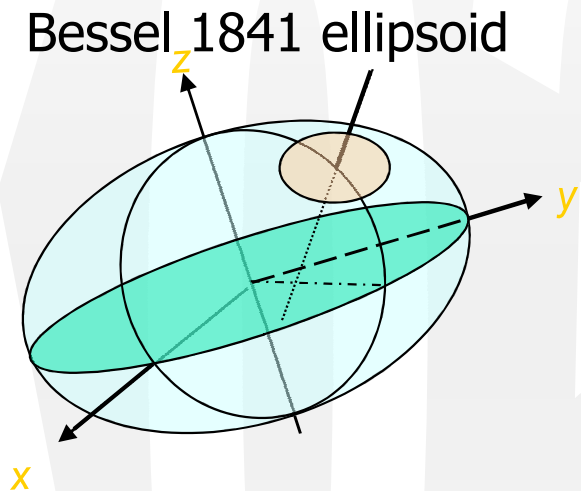
### ~~New national CRS~~



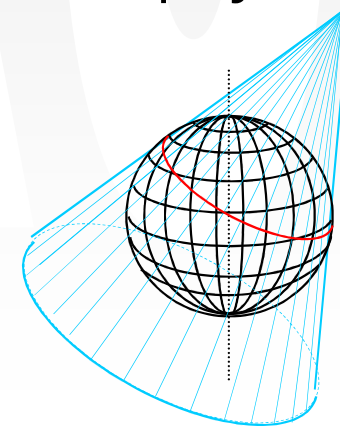


# S-JTSK national CRS in Slovakia

- S-JTSK stands for: System of unified trigonometric cadastre network
- 2D CRS
- Defined in cca. 1919-1920 year
- Definition:
  - Bessel 1841 ellipsoid
  - Krovak projection – oblique conformal conic projection



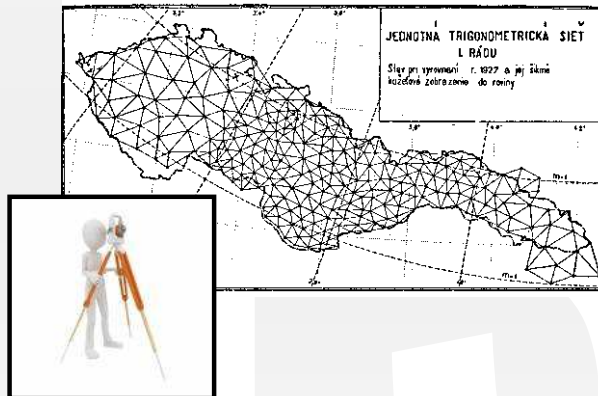
Křovák projection



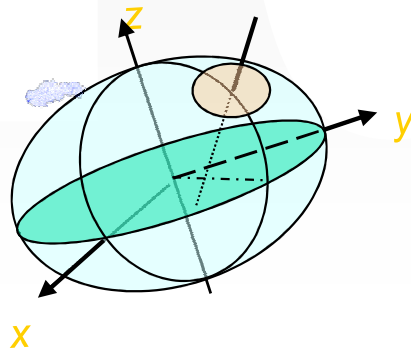
# S-JTSK realisations

## Former JTSK realisation

- Precise angular measurements in Trigonometric network

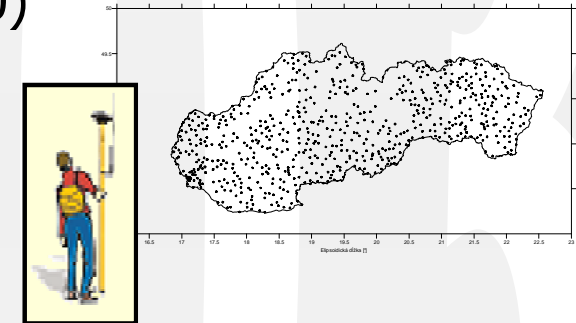


- Adjusted network - fixed to Bessel 1841 ellipsoid through astronomical points + baseline measurement

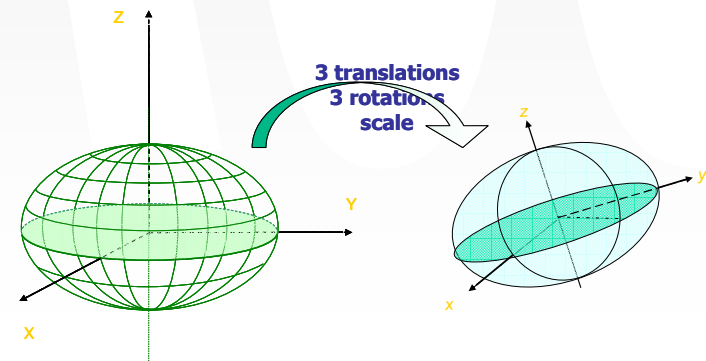


## New JTSK03 realisation

- Precise GNSS measurements in National spatial network (ETRS89 – ETRF2000)



- Adjusted coordinates (network) - fixed to Bessel 1841 ellipsoid from GRS80 ellipsoid through 7 Helmert parameters

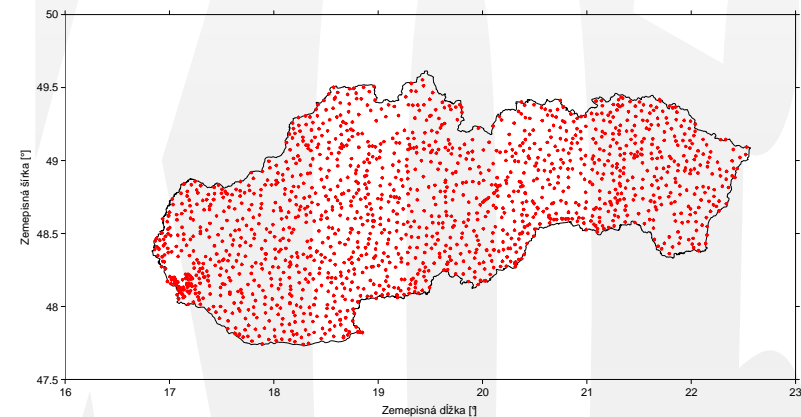




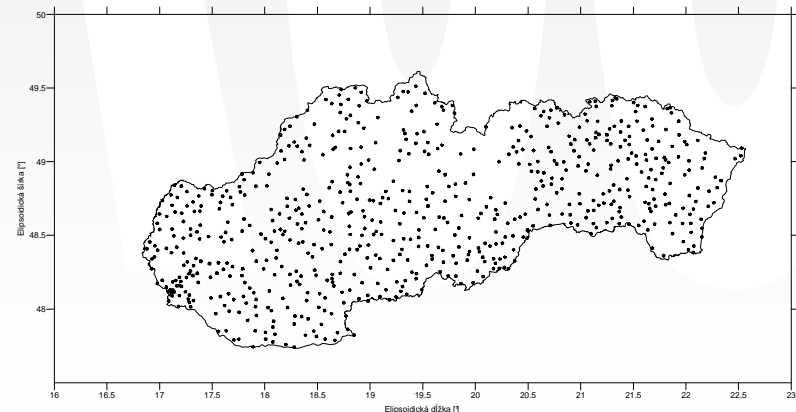
# Relation between S-JTSK (JTSK03) and ETRS89 (ETRF2000)

- ETRS89 is represented in Slovakia by National spatial network (SPS) in SKTRFyyyy (Slovakian terrestrial reference frame)
  - actual version is SKTRF2009 = ETRF2000
    - based on permanent measurements
- All points from National Spatial network (SPS) have ETRS89 coordinates
- 50% of ŠPS C - class have also former JTSK coordinates

## National spatial network (ŠPS) – C class



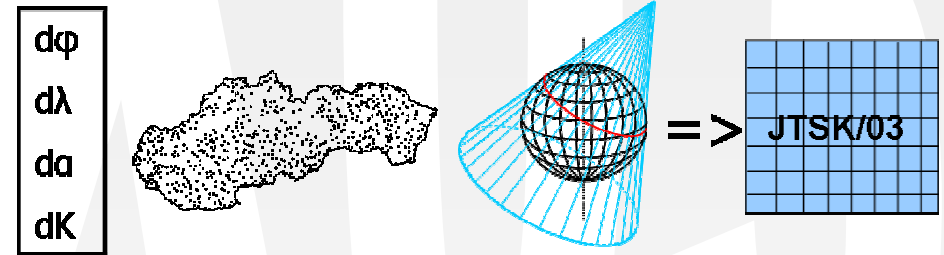
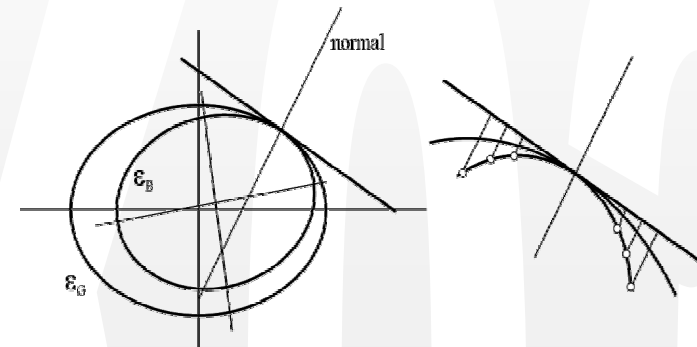
## ŠPS points with former JTSK coordinates



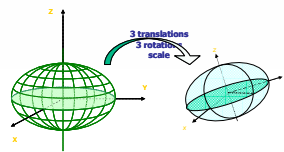


# Determination of relation between ETRS89 and S-JTSK

- 3 step process:
  1. Estimation of 4 transformation parameters – Transformation on ellipsoid surface with coincides normals
  2. Computation of JTSK03 coordinates (usage of 4 transformation parameters)
  3. Estimation of 7 Helmert parameters (3D Helmert transformation) between ETRS89 coordinates and JTSK03 coordinates determined from 4 transformation parameters estimation = estimation **GRS80** ↔ **Bessel 1841** relation



$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}^B = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix} + (1 + s \times 10^{-6}) \cdot \begin{bmatrix} 1 & -r_z & r_y \\ r_z & 1 & -r_x \\ -r_y & r_x & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}^A$$







# JTSK03 implementation to law

- § ■ For legal usage it is important to implement the new realisation to legislative
- First of all for that step you need unique definition about all JTSK03 parts and about procedure of coordinate determination
  - Definition of used ellipsoids
  - Definition of used projection
  - Definition of type and parameters of transformation
  - ...
- Sometimes few problems (usually formal) occur during the process of implementation
- In Slovakia there are:
  - “formal” problem with correct definition of Bessel 1841 ellipsoid,
  - Definition of Helmert transformation reversibility



# Problem with Bessel 1841 ellipsoid correct definition

- different types of Bessel 1841 ellipsoid parameters

query by filter    retrieve by code

Name:

Type:

Area:

North Latitude:  West Longitude:

South Latitude:  East Longitude:

Search    Reset ?

**OGP**

**Ellipsoid [Bessel 1841]**

Code: **EPSG::7004**

Name: **Bessel 1841**

Shape: Ellipsoid

Semi-Major Axis: 6377397.155 [metre](#)

Inverse Flattening: 299.1528128 [unity](#)

4 and values from the length of certain ical Manual;

## Classic literature

Tabulka 1

	Elipsoid			
veličina	Bessel (1841)	Hayford (1910)	Krasovský (1940)	IAG 1967
$a$	6 377 307,155 0 m	6 378 388,000 0 m	6 378 245,000 0 m	6 378 160,000 0 m
$b$	6 356 078,963 3 m	6 356 911,948 1 m	6 356 863,918 8 m	6 355 774,516 1 m
$c$	6 398 786,849 4 m	6 399 836,808 1 m	6 399 698,901 8 m	6 399 617,429 0 m
$f$	1 : 299,152 813 = 0,00334 27731 8158	1 : 297,0 = 0,00336 70033 6700	1 : 298,3 = 0,00335 23298 6926	1 : 298,247 167 = 0,00335 29237 1299
$e^2$	0,00667 43722 3061	0,00672 28700 2233	0,00669 34216 2297	0,00669 48063 2356
$e'^2$	0,00671 82187 9797	0,00676 81701 9722	0,00673 85254 1468	0,00673 97261 2832
$n$	0,00167 41848 0082	0,00168 63406 4081	0,00167 89791 8068	0,00167 92771 0050

**Bessel ellipsoid**

From Wikipedia, the free encyclopedia

The **Bessel ellipsoid** (or **Bessel 1841**) is an important *reference ellipsoid* of continents, but will be replaced in the next decades by modern ellipsoids of continents.

The Bessel ellipsoid was derived 1841 by *Friedrich Wilhelm Bessel*, based on 10 meridional arcs and 38 precise measurements of the Earth in *India*. It is based on 10 meridional arcs and 38 precise measurements of the Earth in *India* in keeping with former *calculation* methods.

**The Bessel and GPS ellipsoids**

The Bessel ellipsoid fits especially well to the *geoid* curvature of Europe and is 700 m shorter than that of the mean Earth ellipsoid derived by satellites.

Below the two axes  $a$ ,  $b$  and the *flattening*  $f = (a - b)/a$ . As for comparison, the parameters of the *GPS* system.

- Bessel ellipsoid 1841 (defined by  $a$  and  $f$ ):
  - $a = 6.377.397,155 \text{ m}$
  - $f = 1 / 299,1528153513233$  (0,003342 773154 ± 0,000005)
  - $b = 6.356.078,963 \text{ m}$
- Earth ellipsoid WGS84 (defined directly by  $a$  and  $f$ ):
  - $a = 6.378.137,0 \text{ m}$
  - $f = 1 / 298,257223563$
  - $b = 6.356.752,30 \text{ m}$

Which parameters are correct?  
Why are they different?

Maybe because in the past, parameters were computed with usage of logarithmic tables – problem with decimals numbers



# Reversibility of Helmert 7-parameter transformation

- Mathematic formula:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^{TARGET} = \begin{pmatrix} T_X \\ T_Y \\ T_Z \end{pmatrix} + (1 + s \cdot 10^{-6}) \cdot \begin{pmatrix} 1 & -R_Z & R_Y \\ R_Z & 1 & -R_X \\ -R_Y & R_X & 1 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^{SOURCE}$$

- Inversion (reverse equation)

- pure mathematic

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^{SOURCE} = (1 + s \cdot 10^{-6})^{-1} \cdot \begin{pmatrix} 1 & -R_Z & R_Y \\ R_Z & 1 & -R_X \\ -R_Y & R_X & 1 \end{pmatrix}^{-1} \cdot \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^{TARGET} - \begin{pmatrix} T_X \\ T_Y \\ T_Z \end{pmatrix}$$

- In literature

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^{SOURCE} = \begin{pmatrix} -T_X \\ -T_Y \\ -T_Z \end{pmatrix} + (1 - s \cdot 10^{-6}) \cdot \begin{pmatrix} 1 & -R_Z & R_Y \\ R_Z & 1 & -R_X \\ -R_Y & R_X & 1 \end{pmatrix}^T \cdot \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^{TARGET}$$

- In software? Nobody knows



# Reversibility of Helmert 7-parameter transformation

- Possible Solutions for reversibility problem
  - Estimation of 2 set of parameters (mathematically not pure)
    - From target to source
    - From source to target
  - 1 set of parameters + mathematic inversion
    - Only few software can apply
  - 1 set of parameters – without inverse possibility
    - All software can apply



# Reversibility of Helmert 7-parameter transformation - Solution in Slovakia

- 1 set of parameters + Web Reference transformation service = AWTs application (Authorized Web Transformation Service)

**Prihlásiť** **TRANSFORMAČNÁ SLUŽBA**  
Geodetický a kartografický ústav Bratislava

## Autorizovaná transformácia súradníc bodov medzi záväznými geodetickými systémami

Na vykonanie autorizovaných transformácií sú použité referenčné digitálne modely reziduálnych zložiek (DMRZ -JTSK), digitálny výškový referenčný model (DVRM-Bpv) a transformačné parametre TPm-JTSK, TPm-JTSKyy.  
Globálne transformačné parametre pre Slovensko z realizácie JTSKyy(Besselov) do realizácie ETRS89(GRS80)  
Popis transformácie : Transformácia medzi 3D karteziánskymi sústavami K1, K2 cez BURŠA-WOLFOV MODEL verzia 1/2007 : 03.10.2006

translacné prvky  $dX = 485,021$  m,  $dY = 169,465$  m,  $dZ = 483,839$  m,  
rotačné prvky  $\omega X = -7,786342''$ ,  $\omega Y = -4,397554''$ ,  $\omega Z = -4,102655''$ ,  
zmena mierky  $K = -0,00000$  ppm.

<http://awts.skgeodesy.sk>

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## Transformácia medzi ETRS89 a S-JTSK

Transformácia na povrchu elipsoidu GRS80 metódou stotožnenia normál vedených ťažiskami

Označenie bodu :

[Ako transformovať jeden bod ...](#)

XX ETRS89 (GRS80)			
X	Y	Z	[m]
B	48	25	16,52652
L	17	40	25,19634
h			526,15
n			43,323
X			
Y			4040964,925
Z			1287590,231
			4748484,723

Nuluj

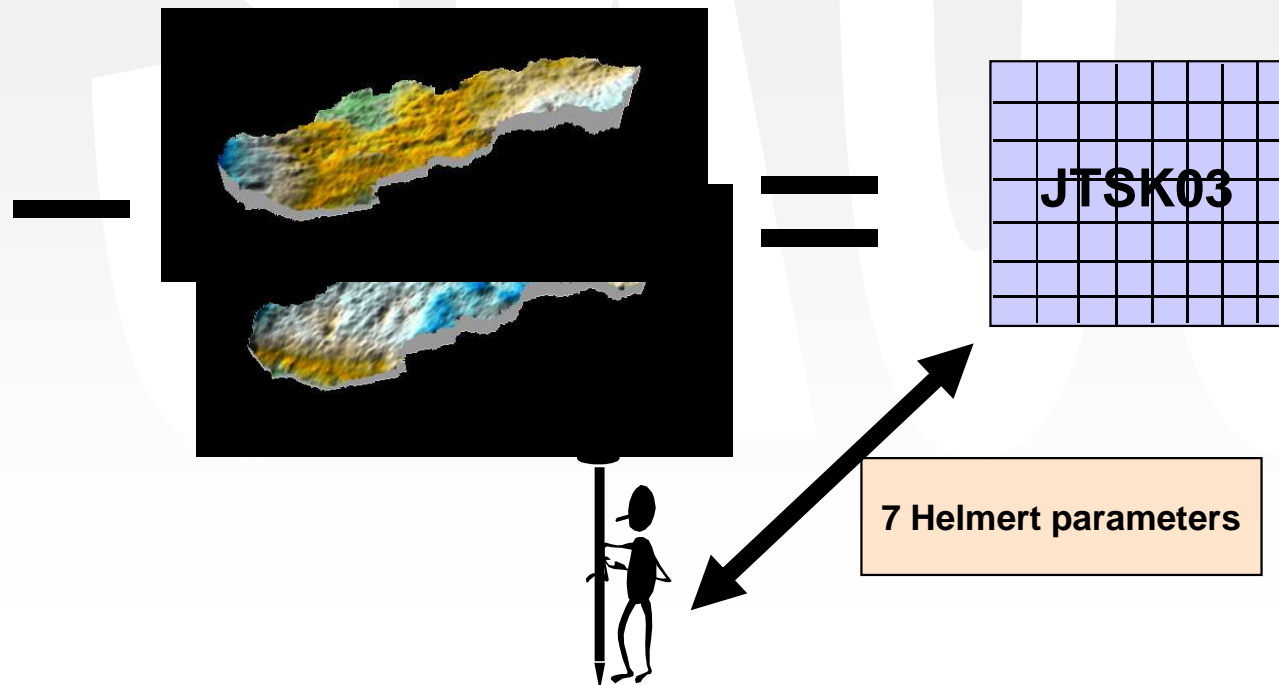
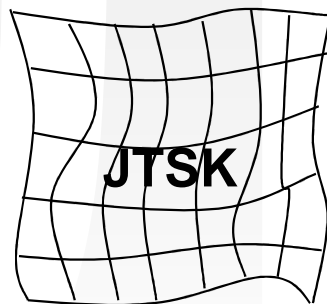
Transformuj

XX S-JTSK (BESSEL)			
X	JTSK	JTSK03	X
x	1254229,966	1254229,900	[m]
y	528936,561	528937,354	[m]
H		482,982	[m] (Bpv)
xx			
X			X
B	482518,255404	482518,255118	[° + '' ]
L	174030,242360	174030,203621	[° + '' ]
h		526,305	[m]



# Relation between JTSK and JTSK03

- Distortions models computed from identical points
- Distortions up to 1.3m
- Separate models for both axes (x,y)
- Grid creation – krigging





# Relation between JTSK and JTSK03

- Possibilities of JTSK – JTSK03 conversion
  - Usage of distortion models
    - Available on AWTS
    - Average global quality – 4cm
      - Sufficient for lot of applications
  - Usage of local transformations
    - Computation of own local transformation parameters





# Recent dilemma in Slovakia

- Reversibility of Helmert transformation
- Hamlet question: Introduced or not introduced the reverse set of 7 Helmert parameters, this is a question?
- Pros
  - Lot of users will be able to get correct JTSK03 or ETRS89 values by their own software not only if they use AWTS
- Cons:
  - 2 set of parameters are not mathematically pure solution







# Conclusion

- Presented example shows that it is not problem to determine the new realisation of any old CRS, but how you can see it is the problem correctly and uniformly to define its relation to nowadays used and recommended systems like ETRS89
- So our recommendation is that if you decide to introduce new set of coordinates for national CRS it will be better to define totally new system not only new frame of old system
- You will avoid complications like it was mentioned in slides above



Thank you for your attention

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